

Semiparametrically Efficient Inference Based on Signs and Ranks for Median Restricted Models

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Abstract

Since the pioneering work of Koenker and Bassett (1978), models involving median and quantile rather than the classical mean or conditional mean concepts have attracted much interest. Contrary to the traditional models where the noise is assumed to have mean zero, median-restricted models enjoy a rich group-invariance structure. In this paper, we exploit this invariance structure in order to obtain semiparametrically efficient inference procedures for these models, based on residual signs and ranks. These procedures include distribution-free hypothesis testing, and root- n consistent estimation. Due to invariance, the test statistics are completely insensitive to possible changes of the underlying density, while the resulting estimators remain root- n consistent, under changes. Yet semiparametric efficiency is attained at correctly specified densities. The testing procedures are improving over sign tests, while estimators improve on LAD estimators. This combination of robustness and efficiency is a definite advantage over classical quasi-likelihood methods. The methods we propose can be applied, without additional technical difficulties, to both cross-sectional and time-series models. They do not require any explicit tangent space calculation nor any projections on these.