Robust estimates for ARMA models

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One of the most successful approaches for robust estimation of ARMA models is based on the robust filter introduced by Masreliez (1975). This approach was used in Denby and Martin (1979), Martin (1979), Martin et al. (1983) and Bianco et al. (1996). The advantage of using a robust filter is that it allows computing the innovations avoiding the propagation of the effect of previous outliers. This property is specially important when working with ARMA(p, q) models with $q > 0$ or when $q = 0$ and p is large. However, we can mention two shortcoming of this approach. The first one is that the resulting estimates are asymptotically biased. The second one is that there is not an asymptotic theory for these estimators, and therefore it is not possible to make inference.

In this talk we propose a new class of robust estimates for ARMA models. To improve robustness the residual innovations are generated using a modified ARMA model where the effect of one innovation on the subsequent periods is bounded. The proposed estimates are a generalization of the MM-estimates introduced by Yohai (1987) for regression.

An stationary and invertible ARMA model can be represented by $\phi(B)y_t = \theta(B)a_t + c$ where a_t is a white noise process , $\phi(B)$ and $\theta(B)$ are polinomials of the form $\phi(B) = 1 - \phi_1 B - ... - \phi_p B^p$ and $\theta(B) = 1 - \theta_1 B - ... - \theta_q B^q$ with roots outside the unit circle. Let $\lambda(B) = \phi^{-1}(B)\theta(B) = 1 + \lambda_1 B + ...\lambda_j B^j + ...$ and consider the $MA(\infty)$ representation of the ARMA process given by

$$y_t = \mu + a_t + \lambda_1 a_{t-1} + ... + \lambda_j a_{t-j} + ... \tag{1}$$

where now $\mu = \phi^{-1}(1)c = E(y_t)$.

The model we are proposing here is of the following form

$$y_t = \mu + a_t + \lambda_1 \sigma\eta(a_{t-1}/\sigma) + ... + \lambda_j \sigma\eta(a_{t-j}/\sigma) + ..., \tag{2}$$

where $\eta(x)$ is an odd bounded and nondecreasing function. Moreover we assume that there exists m such that $\eta(x) = x$ for $|x| \leq m$. Note that in this model the lag effect of a large innovation is bounded and, since $\lambda_j \to 0$ exponentially when $j \to \infty$, the effect of the innovation will disappear in a few periods. This is different of what happen in a regular ARMA model where a very large innovation in period t may have a large influence in many subsequent observations.

If η is the identity $(m = \infty)$, (2) is the equation defining a standard ARMA model. Then we have embedded the ARMA model in a larger family. We call this model, the bounded innovation propagation autoregressive moving average model (BIP-ARMA).We show that the residual innovations for this model are equivalent to those based on robust filters.

To obtain consistency under an ARMA model, we compare a robust scale of the innovations computed under both models: the classic ARMA model, and the modified ARMA model proposed here. We choose the model with smallest scale and the final estimate is a redescending M-estimate with innovations computed according to this model. We find the asymptotic normal distribution of the proposed estimates under an ARMA model. We also perform a Monte Carlo study where we show that the proposed estimates have good efficiency and robustness properties.

References

A. M. Bianco, M. García Ben, E. J. Martínez, and V. J. Yohai (1996). Robust procedures for regression models with ARIMA errors. In *COMPSTAT 96, Proceedings in Computational Statistics*, A. Prat, ed., 27–38, Physica-Verlag.

L. Denby and R.D. Martin (1979). Robust estimation of the first–order autoregressive parameter. *Journal of the American Statistical Association*, **74** , 140–146.

R. D. Martin (1979). Approximate conditional mean type smoothers and interpolators. In *Smoothing Techniques for Curve Estimation*, T. Gasser, and M. Rosenblatt, eds., 117–143, Springer Verlag.

R.D. Martin, A. Samarov and W. Vandaele (1983). Robust methods for ARIMA models. In *Applied Time Series Analysis of Economic Data*, E. Zellner, ed., 153–177, Bureau of the Census.

C. J. Masreliez (1975). Approximate non–gaussian filtering with linear state and observation relations. *IEEE–Transactions on Automatic Control*, AC–20, 107–110.

V.J. Yohai (1987). High breakdown point and high efficiency robust estimates for regression. *Annals of Statistics* **15**, 642-656.