On standard errors of model-based small-area estimators

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Summary

The EURAREA project confirmed the superiority of model-based estimators of small-area means and proportions, with several qualifications, but reported rather disappointing results regarding estimators of their standard errors. We trace this problem to the contradiction between (replicate) sampling from a population, its division to small areas and values of all variables that were fixed, and application of random-effects models which assume that a different population of subjects and a set of small areas are drawn from a hypothetical superpopulation in each replication. The two corresponding perspectives, design-based and model-based, are related by an averaging applied in deriving the standard errors of shrinkage estimators. We regard the design-based perspective as appropriate, but dismiss the standard design-based estimators because they fail to draw on the auxiliary information available in the form of data from other areas, related variables and other surveys.

We show that the model-based estimator of the sampling variance of a small-area estimator is approximately unbiased only when the small-area target is in the typical distance from the national mean or its regression adjustment. Based on this diagnosis, we derive an estimator of the mean squared error (MSE) of the empirical Bayes and composite estimator of the local-area mean in the standard small-area setting. The MSE estimator is a composition of the established estimator based on the conditional expectation of the random deviation associated with the area and a naive estimator of the design-based MSE. Its performance is assessed by simulations in settings that range from the congenial (in close agreement with the assumption) to distinctly uncongenial, exploring the sensitivity of the estimator with respect to some of the model assumptions. Variants of this MSE estimator are explored and some extensions outlined.

Key phrases: *Composite estimation, empirical Bayes estimation, shrinkage, small-area estimation.*

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