ON MEAN SQUARE ERROR OF EBLU PREDICTORS BASED ON THE FORMULA OF ROYALL'S BLU PREDICTOR

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In the paper we consider the problem of the Best Linear Unbiased and the Empirical Best Linear Unbiased Predictors under the general mixed linear model. The BLU predictor was proposed by Henderson (1950) (following Rao (2003)). Formula of the BLU predictor includes unknown elements of the variance-covariance matrix of random variables. If the elements in the formula of the BLU predictor proposed by Henderson (1950) are replaced by some type of estimators, we will obtain the two-stage predictor called the EBLU predictor which is model-unbiased (Kackar and Harville (1981)). Kackar and Harville (1984) gave an approximation to the MSE of the predictor and proposed an estimator of the MSE. The MSE and estimators of the MSE were also studied by Prasad and Rao (1990), Datta and Lahiri (2000), Das, Jiang and Rao (2004).

In the paper we consider the BLU predictor proposed by Royall (1976). Żądło (2004) showed that the BLU predictor proposed by Royall (1976) may be treated as a generalisation of the BLU predictor proposed by Henderson (1950) and proved model unbiasedness of the EBLU predictor based on the formula of the BLU predictor proposed by Royall (1976) under some assumptions. In the paper we derive the formula of approximate MSE of the EBLU predictor and its estimators. We prove that the approximation of the MSE is accurate to terms $o(D^{-1})$ and the estimator of the MSE is approximate unbiased in the sense that its bias is $o(D^{-1})$ under some assumptions, where D is the number of domains. The proof may be treated as a generalization of the results received by Datta and Lahiri (2000). Using our results we present some BLU and EBLU predictors based on special cases of the general linear model and formulae of their MSEs and estimators of their MSEs.

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